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Constraints on Learning in Nonprivileged Domains

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Constraints on learning, rather than being unique to evolutionarily privileged domains, may operate in nonprivileged domains as well. Understanding of the goals that strategies must meet seems to play an especially important role in these domains in constraining the strategies that are generated and in allowing children to evaluate strategies even before they use them. The present experiments showed that children can use their conceptual understanding to accurately evaluate strategies that they not only do not yet use but that are more conceptually advanced than the strategies they do use. In Experiment 1, 5-year-olds who did not yet use the min strategy for adding numbers judged it to be smarter than an equally novel illegitimate strategy, and to be just as smart as their typical strategy of counting from one. In Experiment 2, 9-year-olds who did not yet use the forking strategy to play tic-tac-toe judged it to be even smarter than their own win/block approach. The results demonstrated a large number of commonalities between the functioning of constraints in privileged and nonprivileged domains, as well as suggesting some possible differences.

Recent depictions of the learning of younger and older children present a paradox. The learning of infants and toddlers is often depicted as fast, efficient, and congruent with higher-order domain principles. The learning of school-age children and older individuals, in contrast, is often depicted as slow, inefficient, and superficial. This paradox has been noted previously with regard to portrayals of analogical reasoning (Brown, 1990), but is equally apparent for a host of other topics. This can be seen by contrasting portrayals of early and later learning of mathematical concepts (e.g., Gelman & Gallistel, 1978; Hiebert & Lefevre, 1986), biological concepts (Carey, 1985; Keil, 1989), casual relations (Kuhn, 1990; Leslie, 1984), and spatial layouts (Hermer & Spelke, 1993; Huttenlocher & Newcombe, 1984).

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To explain the rapid cognitive progress of infants and toddlers, a number of investigators have hypothesized that early acquisition of critical concepts is guided by implicit domain-specific principles and constraints (e.g., Carey, 1985; Gallistel, Brown, Carey, Gelman, & Keil, 1991; Gelman & Gallistel, 1978; Keil, 1989; Kellman & Spelke, 1983; Markman, 1989). These principles and constraints are said to provide a base for drawing essential lessons from among the infinite number of conclusions that might logically be drawn from experience. They also are said to distinguish learning in evolutionarily significant domains from learning in nonprivileged domains where such domain-specific principles and constraints are not hypothesized to be present. At times, they also are used to contrast early and later learning within the same domain, as in Gelman and Meck's (1992) account of early and later mathematics learning.

What exactly does it mean to say that learning is guided by principles or constraints? Perhaps the deepest and most comprehensive analysis is that formulated by Gelman and her colleagues (e.g., Gelman, 1993; Gelman & Cohen, 1988; Gelman & Greeno, 1989; Gelman, Meck, & Merkin, 1986). Their analysis emphasizes the following seven properties and effects of principles:

(1) Only a skeletal subset of relevant principles in a domain tends to be present at first, with learning bringing considerable elaboration.

(2) The principles generally begin in implicit form, though they may later become explicit.

(3) The principles of a domain lead children to focus selectively on relevant input and to organize the input in useful ways.

(4) The ways in which principles affect behavior depend not only on conceptual knowledge—the structure of which is given by the principles—but also on procedural and utilizational knowledge—knowledge about how to generate procedures consistent with the principles, and about when such procedures should be applied.

(5) Principles include both domain-specific (necessary) and domain-linked (desirable) properties.

(6) They guide learning of observed procedures and adaptation of known procedures to fit novel task demands.

(7) They allow judgments of the legitimacy of unfamiliar procedures.

Accounts that cite principles and constraints have generally been invoked to explain learning in situations that have three characteristics. First, the domains seem evolutionarily important: object perception (Kellman & Spelke, 1983), face recognition (Morton & Johnson, 1991), syntax (Chomsky, 1965; Johnson & Newport, 1989), vocabulary acquisition (Clark, 1983; Markman, 1989), biology (Carey, 1985; Keil, 1989), mechanics (Spelke, Breinlinger, Macomber, & Jacobson, 1992), number

(Gelman & Gallistel, 1978; Wynn, 1992), mind (Leslie, 1987), and so on. Second, the task of learning about the domains is sufficiently demanding and people's performance sufficiently good that it is plausible to postulate domain-specific learning mechanisms that create specialized representations of that type of information. Third, learning begins sufficiently early, progresses sufficiently rapidly, and occurs sufficiently close to universally that it is plausible to postulate a specific innate basis for knowledge acquisition.

The learning produced in areas in which such principles and constraints are hypothesized is often contrasted with the learning said to be produced by domain-general learning mechanisms in nonprivileged areas (or by special populations, such as Down Syndrome or autistic children, who may lack some of the principles). Relative to the learning guided by domain-specific principles, learning hypothesized to be produced through general-purpose mechanisms is said to be slow, piece-meal, dependent on facilitative structuring of input, and dependent on extensive trial and error. Although a wide range of domain-general mechanisms are acknowledged, the specific contrasts drawn are usually between learning aided by domain-specific principles on the one hand and learning produced by rote association on the other (e.g., Gallistel, Brown, Carey, Gelman, & Keil, 1991; Gelman & Cohen, 1988; Keil, 1990). For example, Gelman and Cohen (1988) contrasted typical and Down syndrome children's ability to adapt to novel counting conditions. The typical children's greater flexibility and effectiveness in dealing with the novel counting tasks was attributed to the guidance conferred by their counting principles, whereas the Down syndrome children's less flexible and effective performance was described as consistent with a rote-associative learning mechanism (e.g., p. 90).

The central argument underlying the present research is not that these analyses of the facilitative role of principles and constraints are wrong, but rather that these concepts have broader applicability for understanding learning than has generally been realized. Just as children's learning in privileged domains may be facilitated by knowledge of the basic principles of the domain, so may much of their learning in altogether mundane domains of no special evolutionary importance. The learning may differ in how early the higher-order knowledge begins to operate and in how universally it operates, but the way in which knowledge of goals and causal structure influences learning may be highly similar.

From this perspective, the relevant contrast is not between domain-specific mechanisms producing fast, efficient, principled learning and domain-general mechanisms producing slow, inefficient, nonprincipled learning. Instead, the critical distinction is between the learning that occurs in contexts in which goals and causal relations are understood and the learning that occurs in contexts in which they are not, regardless of the mechanism that produced the original understanding. When goals and causal relations are understood, learning takes on the characteristics that have been identified with domain-specific learning mechanisms. When such goals and relations are not understood, learning proceeds in the slow, inefficient way that has been identified with general learning mechanisms. From this perspective, the slow and superficial learning that school-age children sometimes show is a product of their not understanding goals and causal relations in the particular domains that have been the focus of attention, rather than an effect of the learning being produced by general learning mechanisms (see Gelman 1993, and Keil, 1990, for related arguments).[1]

The experiments in this article were designed to obtain one of the strongest types of evidence that has been produced for the existence of principles and constraints in evolutionarily significant domains: Ability to accurately evaluate novel procedures. In counting (Briars & Siegler, 1984; Gelman & Meck, 1983; Frye, Braisby, Lowe, Maroudas, & Nicholls, 1989), syntax (Johnson & Newport, 1989; Patkowski, 1980), and other privileged domains, ability to judge the appropriateness of novel forms has been a key source of evidence for claims that children's learning is guided by domain-specific principles. The present experiments demonstrate that understanding of goal structures often allows children to make similarly accurate judgments of novel strategies in domains where no domain-specific learning mechanisms seem plausible. This is shown initially in the context of particular arithmetic strategies and later in the context of strategies for playing tic-tac-toe. We also describe a mechanism, the goal sketch, that uses goal structures to generate and evaluate potential new procedures, and discuss how it might constrain strategy generation.

EXPERIMENT 1

Evaluation of Novel Arithmetic Strategies

By the time they enter kindergarten, most children use multiple strategies to solve addition problems (Geary & Burlingham-Dubree, 1989;

[1] Many terms have been used to refer to the type of meaningful knowledge that can influence the acquisition and execution of strategies: constraints, principles, conceptual knowledge, conceptual understanding, understanding of goals, and understanding of causal relations, among them. These constructs have much in common. They also differ in complex and subtle ways, but analyzing these differences is beyond the scope of the present article. In the General Discussion, the construct focused on in the present article, the goal sketch, is distinguished from all of these related ideas on the basis of its greater emphasis on the goals that useful procedures must meet.

Siegler & Shrager, 1984). Their single most common approach is the *sum strategy*, where they typically first count out each addend on their fingers and then count all of the raised fingers to get the sum. For example, to solve 2 + 3, a child would count, "1, 2, . . . 1, 2, 3, . . . 1, 2, 3, 4, 5." By the middle of first grade, most children discover the *min strategy*, where they represent the larger addend by simply saying it and then count from it the number of times indicated by the smaller addend. For 2 + 3, a child would count "3, 4, 5" or "4, 5." If executed correctly, either strategy will generate the correct answer. The min strategy, however, requires less counting, which leads to faster and more accurate solutions. It also is more advanced conceptually, since it requires the knowledge that a number's magnitude can be represented by simply stating the number and also requires some version of the commutative understanding that changing the order of the addends does not change the magnitude (Fuson, 1988; Resnick & Ford, 1981).

Siegler and Jenkins (1989) used a microgenetic procedure to study how children discover the min strategy. They identified 4- and 5-year-old preschoolers who knew how to add via the sum strategy but who did not yet know the min strategy. Over an 11-week period, these children were presented roughly 30 sessions of experience solving addition problems. Because the combination of videotaped records of ongoing behavior and immediately retrospective verbal reports yields valid trial-by-trial assessments of strategy use in this domain (Siegler, 1987; 1989), this microgenetic method allowed identification of the first trial on which each child used the new strategy and analysis of the behavior that led to the discovery.

If the 4- and 5-year-olds discovered the min strategy in anything other than a highly constrained way, the period leading up to the discovery should have been marked by at least occasional, and probably frequent, use of procedures that violated the requisites for legal addition strategies. In particular, the children would have been expected to produce strategies that were superficially similar, but fundamentally flawed, variations on strategies they had used previously. For example, they might have counted one of the addends twice or started counting from the larger addend and counted up the number of times indicated by the second addend—regardless of whether it was larger or smaller. However, no child was observed to use these or any other illegitimate strategy on even a single trial. Instead, all children produced the min strategy correctly without having tried any illegitimate approach. These results argued for the view that some type of higher order understanding constrained the children's discovery of the min strategy.

The higher-order knowledge was not necessarily explicit. Some of the children showed conscious, explicit, understanding of the min strategy when they discovered it, but almost half showed little if any explicit understanding at that time. This latter group of children gave confused or contradictory descriptions of what they had done on the trial on which they first used the new strategy and did not show any apparent insight into why it might be useful.

These findings led Siegler and Jenkins to hypothesize that children's discovery of the min strategy was constrained by a *goal sketch*. Goal sketches are knowledge structures that embody the hierarchy of subgoals that legitimate strategies in a domain must meet. For example, the goal sketch for addition that was hypothesized by Siegler and Jenkins (1989) indicated that legitimate addition strategies must satisfy the goals of representing each addend (for example by putting up fingers) and quantifying the representations so as to yield a single number to represent the number of objects in the combined sets (for example by counting the raised fingers and using the final count to represent the total number of objects).

Goal sketches do not exist in a vacuum. The goal sketch for a given domain will invariably be rooted in understandings of other domains. For example, the goal sketch for arithmetic presupposes a principled understanding of counting; such strategies as counting from one and counting from the larger addend only make sense if there is a one-to-one correspondence between the numbers being said and the objects being counted, if any set of objects can be counted, and if the final number in the count denotes the cardinality of the set. Application of goal sketches similarly implies the kind of planning competence described by Greeno, Riley, and Gelman (1984) using the planning net formalism. Without such planning ability, it would be impossible to use a goal sketch to judge whether a potential strategy is feasible and whether it meets the principles of the domain.

Although goal sketches are hypothesized to be produced by general learning mechanisms and to be present in a wide variety of domains, they also are hypothesized to possess many of the same properties and serve many of the same functions as principles and constraints in evolutionarily privileged domains. In particular, they are hypothesized to lead learners to focus selectively on relevant input, to include both necessary and optional information, to be built up gradually from an initial skeletal base, to often begin in implicit form, to guide learning of observed procedures and adaptation of procedures to fit novel task demands, and to allow judgments of unfamiliar procedures.

These last two properties seem especially important in the context of strategy discovery. For the 4- and 5-year-olds in Siegler and Jenkins (1989) to have generated the min strategy without ever having tried illegal addition strategies suggested that some type of higher-level conceptual knowledge of addition guided the strategy formation process. Results of

Jones and VanLehn's (1991) GIPS simulation of discovery of the min strategy lent additional credence to this view. GIPS' lack of conceptual understanding of addition allowed it to generate illegal strategies, which would have been executed and would have led to learning of the illegal strategies save for the intervention of the operator of the simulation (Jones & VanLehn, 1991, p. 362).

This difference between the children's and the simulation's behavior suggested that the preschoolers, unlike GIPS, possessed conceptual knowledge that constrained their strategy discoveries. However, it is difficult to base firm conclusions on what children were not observed to do or on the inadequacies of a particular computer program. What was needed to test the goal sketch hypothesis rigorously was a more direct measure of the children's conceptual understanding. Judgments of novel procedures provided such a test. If children possess a goal sketch in a domain, and that goal-sketch guides their evaluations of potential strategies, they should be able to recognize that legal strategies that they do not yet use are superior to illegal strategies that they also do not use.

In Experiment 1, we tested this prediction in the domain where goal sketches were originally hypothesized—preschoolers' addition. The experiment included a strategy-use session and a strategy-judgment session. In the strategy-use session, 5-year-olds were presented a set of addition problems; the purpose was to divide the sample into those who already used the min strategy (the *min group*) and those who did not (the *no-min group*). Children in both groups knew how to add via the sum strategy, but only those in the min group used the min strategy as well. In the strategy-judgment session, the experimenter solved addition problems via three strategies: the sum strategy, the min strategy, and an illegitimate strategy in which the first addend was counted twice (See Table 1). On each trial, children needed to judge the demonstrated strategy as "very smart," "kind of smart," or "not so smart."

Children who used both the sum and the min strategy would be expected to judge both as smarter than the illegitimate approach regardless of whether they judged in terms of how well each strategy met the goals of addition, or in terms of simple familiarity. Their judgments served mainly as a point of comparison for the data of more direct interest—judgments of children who did not yet use the min strategy. These children might judge the relative smartness of the strategies in any of four ways:

1. Familiarity. If children judged on the basis of familiarity, as might be expected if only rote associative mechanisms operated in this domain, the children would presumably prefer novel strategies to the extent that they shared features with familiar approaches. As Table 1 illustrates, the illegitimate and sum strategies share four nonessential features but not the

TABLE 1
The Three Demonstrated Addition Strategies

	Sum	Min	Illegitimate
Observed counting for 6 + 8	1. Count out 6 chips 2. Count out 8 chips 3. Count 14 chips	1. Count out 6 chips 2. Count from 8 to 14	1. Count out 6 chips 2. Count out 6 chips 3. Count 12 chips
Features of the sum strategy			
Both addends are represented when counting out the sum	Yes	Yes	No
Three separate counts are made before the answer is obtained	Yes	No	Yes
Counting always starts at "1"	Yes	No	Yes
When counting to the sum, the first-mentioned addend is always counted first	Yes	No	Yes
The sum count follows representation of the addends	Yes	No	Yes

Note. Only the first feature is necessary for a legitimate addition strategy.

essential feature of both addends being represented. In contrast, the min and sum strategies have in common the single essential feature but none of the other four. Thus, if children who did not yet use the min strategy judged the intelligence of a strategy in terms of its familiarity, and did not accord any special status to the goal of representing both addends, they would judge the sum strategy as the smartest, the superficially similar illegitimate strategy as the next smartest, and the min strategy as the least smart.

2. *Goal satisfaction and familiarity*. A second possibility was that these children would accord special status to satisfying the goal of representing both addends, but would also favor familiar strategies over unfamiliar ones. If so, they would judge the sum strategy as most intelligent, the min strategy as next most intelligent, and the illegitimate strategy as least intelligent.

3. *Goal satisfaction alone*. A third possibility was that children who did not yet use the min procedure would judge the strategies solely on the basis of their fit to the goal sketch and ignore the familiarity of the strategy. This would lead them to judge the sum and min strategies as equally intelligent and both as smarter than the illegitimate approach.

4. *Goal satisfaction and efficiency*. Both the min and sum strategies meet the goals of addition, but the min strategy requires less counting. If children's evaluations are based on efficiency as well as attainment of essential goals, they should judge the unfamiliar min strategy as being

even more intelligent than the sum strategy, with both again being judged smarter than the illegitimate strategy.

Our basic prediction was that the results would conform to one of the latter two patterns rather than to one of the former two. That is, children should judge the strategies on the basis of how well they satisfy the essential, and perhaps the desirable, goals of the domain, rather than on the basis of familiarity. This would provide the same type of demonstration of understanding of principles of the domain as provided in previous studies of counting and syntax.

In a critical way, the present judgment task would be an even more stringent test of understanding than previously used ones. Subjects in the prior studies of counting and syntax needed to evaluate novel procedures that were at the same conceptual level as their existing approaches. For example, needing to label the leftmost object in a row as "3" imposes extra planning and working memory demands, but does not require conceptual competence beyond that hypothesized to be implicit in preschoolers' standard counting. In contrast, the min strategy is generally recognized to demand conceptual understanding beyond that of the earlier-developing sum approach—the understanding that the magnitude of the larger addend can be represented by simply saying the number corresponding to that addend, and that the magnitude of the answer is unaffected by reversing the addend order (Fuson, 1988). Thus, the children in Experiment 1 who did not yet use the min strategy, but who did use the sum approach, were being asked to display understanding beyond that implicit in their addition performance. For them to recognize the appropriateness of the min strategy would provide particularly clear evidence of their understanding of what constituted a legitimate addition strategy.

Method

Participants. Children were 23 kindergartners (M = 5.45 years old, SD = .26 years) who attended a university-based preschool or a university-based daycare center in Pittsburgh. Each child participated in two sessions of roughly 10 min apiece: The strategy-judgment session and the strategy-use session.

Strategy-judgment session. In the strategy-judgment session, children saw the experimenter solve nine problems. Each of three strategies was used to solve one of every successive group of three problems. All children received the same ordering of problems and strategies.

At the beginning of the session, the experimenter sat across the table from the child, with a pile of chips between them, and said:

> I've been all over Pittsburgh, and I've seen lots of different ways that kids add up numbers. What I'm going to do is use these chips here to show you some of the ways that different kids added. Now, sometimes what they did was very smart, sometimes what they did was kind of smart, and sometimes what they did was not so smart at all. The numbers these kids added up were pretty hard, so I don't expect you to know what the right answer is. I just want you to look carefully at the

way they got their answers, and tell me if what they did was very smart, kind of smart, or not so smart at all.

Each trial began with the experimenter showing the preschooler the problem written on a card and reading it aloud (e.g., "6 + 8"). The card was then placed on the table, so the child could see it throughout the trial. When demonstrating the sum strategy (e.g., on 6 + 8), the experimenter said: "First I'll count out both numbers. The first number is 6; 1, 2, 3, 4, 5, 6 (putting out 6 chips). The second number is 8; 1, 2, 3, 4, 5, 6, 7, 8 (putting out 8 chips). Now I'll count up these 6 chips and these 8 to see what 6 + 8 is; 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14. The answer is 14."

When demonstrating the min strategy, the experimenter said, "First I'll count out the smaller number. The smaller number is 6; 1, 2, 3, 4, 5, 6 (putting out 6 chips). Now, I'll start counting from the bigger number, which is 8, and count up the 6 chips to see what 6 + 8 is; 8, 9, 10, 11, 12, 13, 14. The answer is 14."

When demonstrating the illegal strategy the experimenter said, "First I'll count out the first number two times. The first number is 6; 1, 2, 3, 4, 5, 6 (putting out 6 chips). Now I'll count out the first number again; 1, 2, 3, 4, 5, 6 (putting out 6 chips). Now I'll count up these 6 chips with these 6 chips to see what 6 + 8 is; 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12. The answer is 12."

After demonstrating each strategy, the experimenter asked, "Was the way that kid added up the numbers very smart, kind of smart, or not so smart at all?" When the subject had judged the strategy, the experimenter asked, "Why was the way that kid added very smart/kind of smart/not so smart?" If the subject mentioned some part of the procedure but did not indicate why it was smart, the experimenter asked a follow-up question. For example, if the response to the initial question was "The counting was very smart," the experimenter asked "What about the counting made it very smart?"

As shown in Table 1, the illegitimate strategy generates an incorrect answer, while the sum and min strategies generate the correct answer. This raised the possibility that children might use the correctness of the answer, rather than the procedural adequacy of the strategy, as the basis of their strategy judgments. To minimize this possibility, we selected problems that were large enough (sums of 11–15, addend sizes of 5–8) that the 5-year-old subjects would be unlikely to know their answers. The nine problems chosen were 8 + 7, 6 + 5, 7 + 6, 6 + 8, 7 + 5, 5 + 6, 6 + 7, 5 + 7, and 8 + 6. In a previous study, children of the same age from the same preschool and day care center who were asked to provide the answer to similar-size problems were able to retrieve it on less than 3% of trials (Siegler, 1987). To minimize the related possibility that children might use estimates of the correct answer to evaluate the answer generated by the illegitimate strategy, we selected problems that had differences of only 1 or 2 between the correct answer and the answer generated by the illegitimate strategy.

Examination of children's explanations of their judgments on each trial indicated that these efforts had the desired effect. On only two trials in the entire experiment did a child explain a judgment by saying that the strategy produced the wrong answer. In both cases, when the experimenter asked what the right answer was, the child responded with a number that was far from the correct sum. Further, the illegitimate strategy was judged no more favorably when the answer produced by it was off by one than when it was off by two; presumably, the closer misses would have been preferred if children were estimating the correct answer and then comparing their estimate to the answer generated by the strategy.[2]

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[2] An alternative approach that we pilot tested was to have use of the illegitimate strategy include compensating skipped objects or doubly-counted objects so that it would generate the correct answer. For example, on 6 + 8, the experimenter announced that she was

Strategy-use session. In the strategy-use session, children were asked to solve 10 problems. The problems were of three types: small number problems (1 + 3, 4 + 2, 3 + 2); medium number problems (3 + 5, 6 + 4, 5 + 6, 8 + 4); and large-plus-small problems (6 + 1, 11 + 2, 3 + 12). On this last type of problem, the large differences in addend size gave children who knew the min strategy a particularly strong incentive to use it. Previous research had shown that children use the min strategy most often on problems like these, that is, on problems with a large difference between the addends and with a small minimum addend size (Siegler, 1987). The order of problems was randomized separately for each child.

The experimenter began the strategy-use session by saying, "Today, I'd like you to add up some numbers. You can do it any way you want. You can just remember the answer, you can guess, you can count in your head, you can count on your fingers, you can count with these chips, you can do whatever you want to get the answer." Subsequently, at the outset of each trial, the experimenter presented the child with a problem written on a card, simultaneously reading it aloud. The child could see the problem throughout the trial, and chips were always available to help solve the problems. After a child answered a problem, the experimenter asked how he or she got that answer.

Strategy use on each trial was classified using videotapes of overt behavior and the child's immediately retrospective self-report of what he or she had done. Reliability of the coding was checked by having two coders score strategy use on 100 randomly-selected trials. The two raters' classifications agreed on 93% of trials.

The strategy-use and strategy-judgment sessions were approximately 3 days apart. To provide the most conservative estimate of children's strategy knowledge at the time they judged the smartness of the strategies, the strategy-judgment session was always first. This avoided any possibility of learning between the two sessions resulting in a child knowing a strategy at the time of the strategy-judgment task that the child had not known when the strategy-use task was given.

Results and Discussion

Strategy use. The strategy-use session was intended to identify two groups of children: children who could add and who used the min strategy, and children who could add but did not use the min strategy. As in previous studies with 5-year-olds, the children used a variety of strategies; Table 2 describes the seven strategies that were used and their frequency.

Of the 23 children who were tested, 18 knew how to add, defined as answering correctly on at least 3 of the 10 problems and using a strategy other than guessing at least once. They divided evenly into 9 who used the min strategy at least once (the *min group*) and 9 who never used the min strategy (the *no min group*). The remaining 5 preschoolers gave no evidence of knowing how to add (they answered correctly on only 7% of

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counting out 6 objects and then 6 more, but then double counted two of the 12 objects to generate the answer "14." The children who were pilot tested with this approach, however, immediately noted these counting errors, and consistently rejected the illegitimate counts because of them (despite the correct answers that were generated). Since we were interested in knowledge of arithmetic strategies, rather than counting, we decided not to include such compensating counting errors.

TABLE 2
Children's Addition Strategy Use

Strategies	Typical use of strategy to solve 3 + 5	% Use: Children who used min strategy (n = 9)	% Use: Children who did not use min strategy (n = 9)
1. Min	Counting (sometimes with chips or fingers) "5, 6, 7, 8" or "6, 7, 8"	32	0
2. Short-cut sum	Counting (usually with chips or fingers) "1, 2, 3, 4, 5, 6, 7, 8"	5	23
3. Sum	Counting out 3 chips or fingers, then 5 chips or fingers, then counting them all out	12	46
4. Decomposition	Saying "you can add two 5s and then take away 2, so it's 8"	7	0
5. Finger (chip) recognition	Counting out 3 chips or fingers, then 5 chips or fingers, then saying "8" without further counting	1	1
6. Retrieval	Giving an answer, and explaining "I just knew it" or "I remembered it"	39	16
7. Guess	Giving an answer, and explaining "I just guessed it"	3	11
	Unclassifiable	1	3
	Total	100	100

trials) and therefore were excluded from further analyses (except for one analysis, in which performance of these children proved to be uniquely revealing).

Strategy judgments. The strategy-judgment session provided data on the children's evaluations of the smartness of the min, sum, and illegitimate strategies. Judgments that a strategy was "very smart" were assigned a score of 2, judgments that it was "kind of smart" a score of 1, and judgments that it was "not so smart" a score of 0.

The comparison of greatest interest contrasted the judgments of the three strategies made by children in the no min and min groups. As shown in Fig. 1, the judgments of children who did and did not use the min strategy were very similar. The 5-year-olds in both groups judged the min and sum strategies to be quite smart and the illegal strategy to be less smart. A 2 (Child's strategy use: min or no min) × 3 (Strategy being judged: Sum, min, or illegal) repeated measures ANOVA revealed a significant main effect for demonstrated strategy, $F(2,32) = 9.19$, $p < .001$, and no effect for group or for the interaction of the two variables. Post-hoc analyses (Fisher PLSD < .05) conducted separately for each group

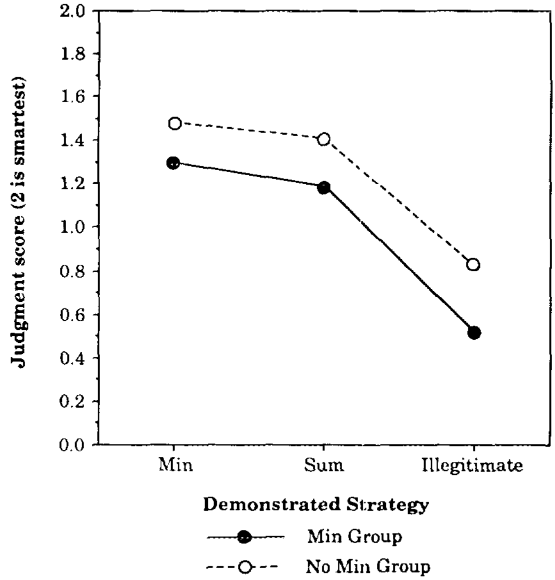

FIG. 1. Mean judgment scores for arithmetic strategies.

indicated that both children who did and children who did not use the min strategy judged the illegal strategy to be significantly less smart than either the min or the sum strategies. Children who did not yet use the min strategy, like those who did, rated the min strategy as slightly smarter than the sum approach, but the difference was not significant for either group.

These patterns characterized the large majority of individual children. Of the children who already used the min strategy, seven judged it to be smarter than the illegitimate approach; the other two judged the two strategies to be equally smart. Of the children who did not themselves use the min strategy, six judged it smarter than the illegitimate strategy, two judged the strategies to be equal, and one child judged the illegitimate strategy to be smarter.

The overall pattern of judgments—sum and min strategies both judged smarter than the illegal strategy and equal to each other—indicates that the 5-year-olds judged the strategies primarily on how well they met the goals of legal strategies in the domain, rather than on the basis of familiarity or efficiency. If children judged on the basis of the strategies' familiarity, those who did not themselves use the min strategy definitely should have judged the familiar sum strategy as smarter than the unfamiliar min strategy. They also would have been expected to judge the illegal strategy, which shared many features with the sum strategy, as smarter than the min strategy, which shared fewer features with it. Instead, children in the no-min group were so unconcerned with familiarity that they judged the unfamiliar min strategy to be as smart as (directionally slightly smarter than) the sum strategy that they themselves often used.

Thus, a central prediction of the goal sketch hypothesis—that children who possess such knowledge should evaluate strategies they do not know on the basis of their understanding of the goal structure of the domain—was supported by children's judgments of arithmetic strategies. Before accepting this interpretation, however, two alternative explanations should be considered. First, because the min strategy involved less counting than the illegitimate strategy, it was possible that children might judge it to be smart not because it fit their goal sketches, but because it took the experimenter less time to execute. Greater efficiency of execution did not lead children to evaluate the min strategy more positively than the sum strategy, but it was possible that children might have viewed both unfamiliar strategies as illegitimate and preferred the min strategy on the logic "Even if it isn't right, at least it's fast." Examination of the data, however, provided no indication that the time taken to demonstrate a strategy was related to children's judgments of its smartness. Within each strategy, correlations between time of execution and judgment of the strategy's smartness were weak (r's $< .2$) and non-significant (p's $> .4$). Further, in a regression analysis in which mean presentation time and type of strategy were used to predict mean judgment score for each strategy, type of strategy was the only significant predictor of judgment score, $F(1,52) = 7.51$; $p < .01$. Length of presentation did not add significant independent variance, partial $F(1,52) = 1.27$. Thus, children's preference for the min strategy over the illegitimate strategy did not derive from the min strategy being faster.

A second alternative explanation was that children's judgments reflected sensitivity to extraneous elements of the instructions or procedure, rather than an evaluation of the fit between the demonstrated strategy and the goals of the domain. The five children whose addition performance was inadequate to be included in the study provided a serendipitous control for this possibility. If the pattern of judgments was attributable to some inadvertent tip-off within our procedure, rather than to knowledge of the goals that legitimate strategies need to meet, the judgments of children who could not add, but who were presented the judgment task before their lack of addition skill was known, should have paralleled the judgments of the children who could add. In fact, these children judged the three strategies to be almost identically smart, and the ordering of their ratings did not resemble that of the children who knew how to add (sum strategy mean of 1.3, illegitimate and min strategy means of 1.1). Thus, it seemed unlikely that the pattern of judgments was attributable to the experimenter or the procedure inadvertently cueing children as to the smartness of the strategies.

Explanations of strategy judgments. After each trial, children were asked to explain their judgments. These explanations were coded as either explicit or vague. Explicit reasons were those that included specific arguments concerning why a strategy was or was not smart. For the min strategy, these were usually responses indicating that counting began at the value of the larger addend. For example, one child, who had just witnessed the experimenter solve 6 + 5 using the min strategy judged it to be very smart, saying: "She didn't have to count up those 6 because she was starting with the number 6." Explicit explanations for the sum strategy were generally those on which children indicated that both addends were represented: "First he counted up the 8 and then he counted up the 7." For the illegitimate strategy, most explicit explanations indicated that only one addend had been used or counted. An example given by a girl who saw 7 + 6 solved with the illegitimate strategy was: "He put 7 and 7, he should have put 7 and 6." Vague responses included those where children replied "I don't know" or where they answered but failed—even after the experimenter's probes—to identify a relevant aspect of the counting (e.g., "I just thought the counting was good" or "I didn't like the adding"). Reliability of the coding of the explanations over 100 trials was 91%.

The results indicated that explicit, verbalizable understanding rarely underlay the 5-year-olds' judgments of the addition strategies. A somewhat greater number of children who themselves used the min strategy provided at least one explicit explanation among their three judgments of the min strategy (4 of the 9 children, vs 0 of 9 in the no min group, χ^2 (df = 1; n = 18) = 5.64; $p < .05$). Even among children who themselves used the min strategy, however, the majority did not provide explicit justifications for their judgments on any of the three min-strategy trials. The numbers of children in the min and no min groups who gave explicit justifications for the other two strategies did not differ. For the sum strategy, three children in the min group gave at least one explicit explanation, compared to four children in the no min group. For the illegitimate strategy, four children from the min group provided explicit explanations, versus six from the no min group. Thus, the difference in explanations of the min strategy did not seem to reflect general differences in explanatory ability. Children's judgments indicated implicit knowledge of the superiority of the min strategy even before they began to use it, but their explanations suggested that ability to verbalize that understanding only came with use of the strategy (and not always very quickly even then).

To summarize, in Experiment 1, children who did not yet use the min strategy nonetheless judged it to be smarter than the illegal strategy and to be at least as smart as their most common approach, the sum strategy. This was especially impressive because the min strategy was more conceptually advanced than the strategies that they themselves used. When judging the efficacy of novel strategies, the 5-year-olds apparently ignored surface similarity to their existing strategies. Their judgments were not based on explicit knowledge about the domain; none of the children who did not use the min strategy, and slightly less than half of those who did, could explain why they thought it smart. Consistent with the goal sketch hypothesis, these results indicate that implicit understanding of the goal structure of addition, together with underlying understanding of counting and planning, was sufficient to allow kindergartners to evaluate unfamiliar addition strategies in reasonable ways.

EXPERIMENT 2

Evaluation of Novel Tic-Tac-Toe Strategies

In Experiment 2, we examined the role of goal sketches in third graders' tic-tac-toe. The experiment was motivated by two issues. One concerned whether the area examined in Experiment 1 was in fact a nonprivileged domain. Although discovery of the min strategy seems like an example of everyday learning, it was possible that acquisition of all addition strategies benefits from domain-specific mechanisms specialized for producing learning about numbers and/or arithmetic. Even infants have basic quantification and addition/subtraction abilities (Antell & Keating, 1983; Starkey, Spelke, & Gelman, 1990; Wynn, 1992); acquisition of the min strategy might be produced through the same, specialized mechanism. Tic-tac-toe, a task where specific evolutionarily based preparation is totally implausible, allowed a cleaner test of the hypothesis that conceptual understanding in nonprivileged domains produces judgments of novel procedures that parallel the judgments in privileged ones.[3]

A second motivation for the experiment was to explore whether goal sketches can include information about desirable, as well as essential,

[3] Even on as culturally and historically specific a task as tic-tac-toe, broadly applicable principles likely play a role, both in facilitating learning and in providing a basis for evaluation of novel strategies. In particular, principles of logic seem likely to have facilitated learning of tic-tac-toe and to have made possible evaluations of the strategies that children were shown. As this example suggests, the frequent dichotomy between domain-specific and domain-general principles is too simple; it masks an underlying continuum of degrees of applicability of principles. Also, as noted in the case of arithmetic, goal sketches specifically relevant to a given task will generally rest on a foundation of other principled understanding. The discussion here focuses on the implausibility of constraining knowledge specific to tic-tac-toe; it is not intended to imply that more generally applicable principles do not provide a foundation for learning and evaluation in this domain as in others.

properties of strategies. In Experiment 1, even children who knew and used the min strategy appeared to judge solely on whether the demonstrated procedure met the requirements for a legal addition strategy. The data provided no evidence that children considered the efficiency with which the strategy met the goals. One might conclude from these data that goal sketches can only produce binary judgments—either the strategy is legal or it is not—rather than producing graded judgments that recognize that some legal strategies are more effective than others. This seems unlikely to be true in general, though. It seems more probable that children at times include desirable as well as essential features within goal sketches. For example, they may evaluate more favorably strategies that are efficient, flexible, or interesting, relative to alternative approaches.

Tic-tac-toe seemed a good task for testing whether children's goal sketches do at times include information about desirable as well as essential properties for strategies to have. The basic goal-sketch for tic-tac-toe was hypothesized to indicate that any acceptable strategy for playing the game must include procedures for meeting two essential goals: winning and not losing. It also seemed likely that experienced players, at least, would include the information that it is desirable to try to win in ways that are difficult to detect and/or stop. This would allow them to judge strategies along a continuum of likelihood of meeting the basic goals.

Previous research indicated that the large majority of first and second graders, and about half of third graders, use a tic-tac-toe strategy that incorporates in a transparent way the essential goals of winning and not losing (Crowley and Siegler, 1993). These children first attempt to identify a move that will produce an immediate win. If a win is not possible, they look to see if they can block a potential win for their opponent on the opponent's next turn. If they can neither win nor block, they attempt to put two Xs in a row so that—if their opponent fails to block—they can win on their own next turn. We will refer to this approach as the *win/block strategy*.

By third grade, about half of children begin to use a more sophisticated approach, the *forking strategy*. This strategy involves trying to create a situation in which it is possible to move to a square that creates two separate winning paths. Even if the opponent blocks one path, the player can win by completing the other. As the game proceeds, the player on each turn considers winning and blocking opportunities as well, first looking for squares where a win is possible, then for squares where a block is needed, and then for possible forks (thus, the approach also could be called the win/block/fork strategy). Using the forking strategy demands looking ahead to anticipate the opponent's optimal next move and then considering the moves that could be made in response. Thus, like the previous analysis of addition strategies, it points to the central role of planning competence in applying goal sketches.

Both the win/block strategy and the forking strategy include the essential features of the goal sketch—they both address the main goals of winning and not losing. However, the forking strategy possesses the additional desirable feature of creating two potential ways of winning and therefore of sometimes allowing the player to defeat opponents who consistently block straightforward efforts to win.

In addition to both the win/block and forking strategies incorporating the basic goals specified within the hypothesized tic-tac-toe goal sketch, indirectly relevant evidence gave reason to suspect that the goal-sketch constrains children's generation of new tic-tac-toe strategies. Crowley and Siegler (1993; Experiment 3) found that even when children were given strong incentives to focus exclusively on winning or exclusively on not losing, they did not create strategies that violate the hypothesized basic goal sketch, for example by focusing exclusively on winning. Instead, they pursued both of the essential goals, but spent more time searching for moves that served the goal that was relevant in the particular situation.

Experiment 2 of the present study was designed to provide more direct evidence that children have goal sketches for tic-tac-toe strategies and that they use them to evaluate strategies that they themselves do not yet use. Its basic logic and procedures closely paralleled those of Experiment 1. First, during the strategy-judgment phase, children observed games where a player pursued either the win/block strategy or a forking strategy; the task was to judge how smart each strategy was. Then, during the strategy-use phase, children's own activities were used to identify those who used the forking strategy (the *fork group*) and those who did not (the *no-fork group*).

As in Experiment 1, the key data were the judgments of children who did not yet use the strategy of interest, in this case the forking strategy. The basic prediction was that despite considerations of familiarity pushing in the opposite direction, children in the no-fork group would rate the forking strategy, which they did not use, as being as smart as or smarter than the win/block strategy, which they did. Such a result would unambiguously indicate that children construct goal sketches in nonprivileged as well as privileged domains, and that such goal sketches can include information about desirable as well as essential properties of strategies.

Method

Participants. Children were 24 third-graders (M = 9.53 years old, SD = .44 years) attending a suburban Pittsburgh public school. Third-graders were chosen as participants because previous experiments indicated that about half would know the forking strategy, while the other half would not know it but would be skillful at using the win/block strategy (Crowley & Siegler, 1993).

Strategy-judgment phase. The experiment was conducted in a single 20-min session, with the first half being the strategy-judgment phase and the second half the strategy-use phase. During the strategy-judgment phase, the children saw a computer program play six games of tic-tac-toe on the screen of a Macintosh SE. In each game, the child was asked to judge the quality of X's approach, which followed either the win/block strategy or the forking strategy. Games were only played up to X's third move (the fifth move overall), so that children could see X set up the potential win(s), but could not see whether X went on to victory.

Before the first game the experimenter said:

> I've been all over Pittsburgh, and I've seen lots of different ways that kids play tic-tac-toe against this computer here. I'm going to show you some of the games they played. In all of these games, the kids played the X's, and in all of these games, the kids were not trying to be nice to the computer, but were trying to win. Sometimes the way they tried to win was very smart, sometimes it was kind of smart, and sometimes it was not so smart at all. You're going to see only the first part of each game. I want you to watch very carefully the way the kids put their X's to try to win. After we see the first part of the game, I'm going to ask you how smart the X's were and who you think will probably go on to win the game.

Children then watched the first game. Before each move, the experimenter said "It's X's (O's) turn now." After the last X move of each trial, children were given five seconds to look at the board, with the experimenter reminding them that they needed to figure out how smart the X's were. The experimenter then asked, "Was the way that the X's tried to win this time very smart, kind of smart, or not so smart at all? Why?" Following the child's answer, the experimenter asked, "Do you think the X's will pretty much win for sure, or do you think that the O's will pretty much win for sure, or do you think that either one might be able to win?" This procedure was followed in all six strategy-judgment games, three in which the win/block strategy was demonstrated and three in which the forking strategy was.

Strategy-use phase. During the strategy-use phase, children played two types of games against the computer. First, in the *full game phase*, play began with an empty board and continued until the child had won, lost, or tied. To allow children maximal opportunity to demonstrate their skills, the child always went first and played the X's.

The *partial game phase* came next. In its beginning, children were told they would be playing a few games that already had some X's and O's on the board, that they would play the games to the end, and that they should try their hardest to win. There were six such games, each of which started with two X's and two O's on the board. In the three potential-win games, the children could win on the first move; in the three potential-fork games, they could not win, or block on the first move, but could create a fork, which guaranteed an opportunity to win on the child's next move. After their first move in each of these games, children were asked why they chose that space. They then finished the game with no further questions.

Including both types of games provided two measures of whether children knew the forking strategy. The full games allowed children to plan and execute the forking strategy in whichever way they usually would; such games offered the advantage of a familiar format. However, because setting up a fork required three moves, children might forget or be distracted from the goal they were pursuing before they achieved it. In the partial games, children needed to make only one move to set up a fork. This significantly reduced the memory load and the likelihood of being distracted before the fork could be set, assured that the program would not inadvertently frustrate the particular fork the child planned to set, and put the child in a position to make a move that would guarantee a win no matter what the computer did. However, it was possible that the unfamiliar task of entering partially played games might increase the difficulty of creating a fork. The two tasks together thus seemed more likely than either alone to reveal any knowledge of forking that children had.

In both the full and partial games, children made their moves by touching the desired square on the computer's touch-sensitive screen. The computer made its moves when the experimenter pressed a key. Its play was governed by the following formula: win if possible; otherwise, block if possible; otherwise, move to a corner opposite an X if possible; otherwise move to an open corner if possible; otherwise, move to the middle if possible; otherwise, move to a side. This strategy made the program vulnerable to forking but otherwise an optimal tic-tac-toe player.

When the child won a game, the winning line of X's flashed three times in conjunction with the computer playing digitized sounds of trumpet fanfares and cartoon characters shouting with glee. When the computer won, the winning line of Os flashed three times, but the computer was silent.

Results and Discussion

Strategy-use phase. All children in the sample knew how to play tic-tac-toe at least at the level of the win/block strategy. In the partial games, every child made the winning move in all of the games where wins were possible on the next move (Table 3). Children were also proficient blockers, with 23 of the 24 playing the computer to a draw in at least one of the full games (the one remaining child defeated the computer by setting forks in two of the three games and therefore was judged likely to be able to block as well). Overall, children played the computer to a draw in 63% of full games and defeated it in an additional 11%.

Almost half of the children, 11 of 24, used the forking strategy in the full games, the partial games, or both. In the full games, children were classified as using the forking strategy if they set a fork and on their next move completed whichever line of three the computer did not block. Because no verbalizations were collected during these games, children's behavior was the only index of their knowledge of forking. In the partial games, children were asked after their first move why they had gone there. If they made a forking move and gave an explanation that indicated they were trying to create two ways to win, they were judged to know the forking strategy. Of the 24 children, 5 showed knowledge of forking in both types of games, 5 in the partial games only, 1 in the full games only,

TABLE 3
Tic-Tac-Toe Performance

	Full games			Partial games	
	% Won	% Lost	% Tied	% Forking moves made	% Winning moves made
Fork group	24	24	52	49	100
No-fork group	0	28	72	0	100

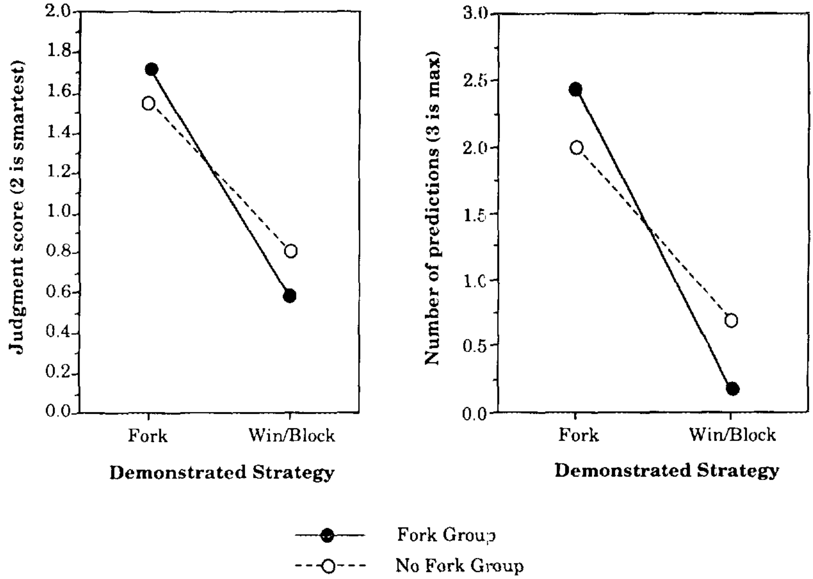

FIG. 2. Mean judgment scores for tic-tac-toe strategies and mean number of predictions that X would win.

and 13 in neither type of game. The 11 children who showed knowledge of forking in one or both types of game were labeled the *fork group;* the other 13 were labeled the *no-fork group.*[4]

Strategy-judgment phase. Children's strategy judgments were scored as in Experiment 1; ratings of "very smart" were assigned 2 points, ratings of "kind of smart" 1 point, and ratings of "not so smart" 0 points. A 2 (Child's strategy use: Fork or no fork) × 2 (Strategy being judged: Fork or win/block) repeated measures ANOVA conducted on the judgment scores revealed a significant main effect for demonstrated strategy, $F(1,22) = 96.85$, $p < .001$, and a marginally significant interaction between the child's strategy use and the strategy being judged $F(1,22) = 3.93$, $p = .06$. Both groups of children judged the forking strategy to be smarter than setting up a single way to win (Figure 2, left panel). The judgments of children who knew how to fork were somewhat more extreme; they judged the forking strategy a little smarter and the win/block

[4] Within the partial games, where both move and explanation data were obtained, the two measures yielded almost identical results. There were only two trials where children made the correct forking move without explicitly stating that they were trying to fork. In both cases, the children on other trials in the partial games set forks and stated that forking was their goal. Therefore, both children were included in the forking group. This meant that all classifications of children into forking and non-forking groups would have been the same if the criterion for forking in the partial games had been simply setting one or more forks.

strategy a little less smart than did children who did not know how to fork, but neither difference between fork and no-fork groups was significant.

Analysis of children's predictions of who would win each game (Figure 2, right side), yielded similar results. A 2 (Child's strategy use: Fork or no fork) $\times$ 2 (Strategy being judged: Fork or win/block) repeated measures ANOVA on the number of times children predicted X would win revealed a main effect for the strategy being judged, $F(1,22) = 46.26$, $p < .001$, and a marginally significant interaction between children's strategy use and the strategy being judged, $F(1,22) = 3.49$, $p = .08$. Children in both groups thought that the forking strategy was more likely to produce a win (rather than a tie or a loss), though the difference in predictions regarding the two strategies' likelihood of producing a win was somewhat greater among children who themselves used the forking strategy.

These patterns were quite consistent across individual children. All 11 children who themselves used the forking strategy judged it to be smarter than setting up a single way to win. Of the 13 children who did not use the forking strategy, 11 judged forking to be smarter and 2 judged it to be equally smart as setting up a single potential winning path. Similarly, all 11 children who used the forking strategy predicted more X wins when the X player created a fork. Of the 13 children who did not use the strategy, 8 predicted more X wins in forking games, 3 predicted equal numbers of X wins for the two types of games, and 2 predicted that X would win more often in games with a single winning path.

As in Experiment 1, we coded children's explanations of their judgments as explicit or vague. For the forking strategy, explanations were coded as explicit if children indicated that the X's had set a fork. One example was: "He put them (the X's) so that he could win two ways and the O can only block him one way, so either way the X's win." For the win/block strategy, explanations were classified as explicit if children indicated that X had only one way to win. One example was: "This kid isn't trying to trick the computer, he's just putting X's in a row here, but the Os can block him." Explanations were coded as vague if children gave responses such as "I just liked the way the X's played" or "I don't know." Reliability of the coding, measured over 100 trials, was 93%.

Children in both groups were able to provide explicit justifications for most of their judgments. All 24 children in the two groups provided at least one explicit justification for the forking strategy. For the win/block strategy, 23 of the 24 children provided at least one explicit justification (all but one in the no-fork group). Thus, both children who did not themselves set forks and children who did judged forking to be a smarter strategy than setting up a single winning opportunity and also were able to provide explicit rationales for why it was superior.

GENERAL DISCUSSION

The present experiments had their origins in a simple question: How is it possible for children to discover legitimate strategies without ever trying illegitimate ones? Siegler and Jenkins (1989) proposed that children can sometimes do this because they possess goal sketches, abstract specifications of the goals that legitimate strategies in a domain must meet, which they use to evaluate potential strategies before they try them. The two experiments in the present study tested whether children can in fact accurately evaluate strategies before they begin to use them. Experiment 1 focused on the domain in which the goal sketch hypothesis was first proposed, children's addition. We found that 5-year-olds who did not yet use the min strategy nonetheless judged it to be smarter than an equally novel illegitimate strategy and to be at least as smart as their own most frequent approach, the sum strategy. Experiment 2 showed that children's ability to accurately evaluate novel strategies was not limited to the domain of addition or to preschoolers. Third graders who did not use the forking strategy for tic-tac-toe judged it to be even smarter than the win/block strategy they did use.

The results extended previous findings regarding judgments of novel procedures in at least four ways. First, they showed that ability to evaluate unfamiliar strategies accurately was not unique to evolutionarily privileged domains; this point was especially clear in the Experiment 2 evaluations of tic-tac-toe strategies. Second, they indicated that such accurate evaluations were not limited to strategies at the same conceptual level as the strategies children themselves used; both the min strategy and the forking strategy demanded conceptual understanding beyond that required by the strategies the children used. Third, the results demonstrated that such judgments were not limited to a simple dichotomous discrimination of legitimate from illegitimate approaches, but rather reflected understanding of desirable as well as necessary features; this was especially clear in the tic-tac-toe results, where the entirely legitimate win/block strategy was judged as fairly smart, but the forking strategy was judged as smarter. Fourth, the results showed that judgments of children who did not yet use a conceptually-advanced strategy closely paralleled the judgments of children who already used it; this was evident for both addition and tic-tac-toe.

The results also had a number of more general implications for understanding similarities and differences between present ideas about structures such as goal sketches, that constrain learning in nonprivileged domains, and previous proposals regarding domain-specific principles and constraints that serve a similar function in domains of likely evolutionary importance. In this concluding section, we compare and contrast the two types of constraints.

Similarities between Constraints in Privileged and Nonprivileged Domains

The present research, combined with previous studies, provides evidence for an intriguing claim made by Gelman (1993). The claim was that well-organized knowledge in entirely mundane domains may constrain learning in the same way as principles and constraints that operate in evolutionarily privileged domains. That is, despite differences in the domains being learned, the constraining effects of well-organized knowledge may be similar.

Evidence supporting this hypothesis can be seen by examining the applicability to tic-tac-toe, clearly a task without any specific evolutionary preadaptations, of the seven properties and functions of principled knowledge described by Gelman and her colleagues in the context of domains believed to be evolutionarily privileged, such as counting and syntax.

(*1*) *Initial skeletal understanding with later elaboration.* Findings from Crowley and Siegler (1993, Experiment 1), that most 5-year-olds have knowledge of winning but not blocking, most 6-year-olds of winning and blocking but not forking, and roughly half of 9-year-olds of winning, blocking, and forking, provided evidence for this type of progression.

(*2*) *Initial implicit understanding often forms a base for later explicit understanding.* The present Experiment 2 provided clear evidence that by age 9, children have explicit knowledge of why certain tic-tac-toe strategies are effective. Preliminary results of an ongoing study suggest that this knowledge began in implicit form. Thus far in this study, 14 kindergartners who themselves used the win/block strategy have observed a computer program play tic-tac-toe. Half have seen the computer play the win/block strategy that all of these children already knew; the other half have seen it play the forking strategy that none of them knew. In the games the children saw, both strategies always led to wins for the X's. Despite these similar outcomes, children who observed the forking strategy have rated the computer significantly smarter than have children who observed it playing the win/block strategy, $t(12) = 2.49, p < .05$. Only 1 of the 7 children who observed the forking strategy, however, has been able to say why it is smart, despite the entire group rating it very smart in absolute terms (a mean of 1.82 on the same 0–2 scale used in Experiment 2 of the present article). The difference between these kindergartners' implicit understanding of the smartness of the forking strategy and the Experiment 2 third graders' explicit understanding of it suggests that conceptual understanding may often proceed from implicit to explicit in nonprivileged as well as privileged domains.

(3) *Focus on relevant input, and useful organization of that input.* In the present Experiment 2, when children who did not themselves use the forking strategy saw the experimenter execute it, they were able to explicitly identify those aspects that made it preferable to their own win-block approach. This indicated that they had focused on and effectively represented the key parts of what they had seen.

(4) *Presence of conceptual, procedural, and utilizational knowledge.* Ability to recognize the intelligence of the forking strategy that they did not use attested to the children's conceptual understanding of tic-tac-toe. Ability to effectively play tic-tac-toe attested to their procedural competence. Evidence of utilizational knowledge came from a previous study, Crowley and Siegler (1993, Experiment 3). In this study, children were told either that winning was the most important goal, that not losing was the most important, or that the two goals were equally important. Children responded to the instructions by spending more time searching for wins (and more consistently detecting potential winning moves) when that goal was emphasized, and more time searching for blocks (and more consistently detecting potential blocking moves) when that goal was highlighted.

(5) *Inclusion of both essential and optional properties.* Attesting to this property, children in Experiment 2 of the present study indicated that the forking strategy was superior because it included the essential feature of not allowing the opponent to win and also the desirable feature of creating two potential winning moves for oneself.

(6) *Principles guide adaptation of known procedures to novel task demands.* The evidence here is the same as for the utilizational knowledge in Point 2. The novel task of being told by an adult that in this game the essential goal was either to win or not lose led children to alter their searches for winning and blocking moves in ways that helped meet the unusual task demands.

(7) *Principles allow judgments of the legitimacy of novel strategies.* The present Experiment 2 provided clear evidence that children's conceptual understanding of tic-tac-toe allows them to accurately judge strategies that they themselves do not use.

Together, these results indicate that organized understanding in nonprivileged domains such as tic-tac-toe can produce effects that parallel those of organized understanding in evolutionarily privileged domains. This suggests a potential answer to the paradox raised at the outset of this paper. The key proximal determinant of the efficiency and effectiveness of learning seems to be neither age nor the evolutionary status of the domain nor whether original learning was produced by domain-specific or general learning mechanisms. Rather, the key determinant of learning seems to be the quality of organization of the underlying knowledge base, with quality of organization defined in terms of how well the knowledge base reflects the essential structure, causal relations, and goals of the domain.

What type of information might a well-organized knowledge base contain? Conceptual, procedural, and utilizational knowledge all seem central (Greeno et al., 1984). For example, a well-organized tic-tac-toe knowledge base would seem likely to include the information that the main goals of the game are to win and not to lose (in that order); that there are a number of strategies such as the win/block and forking approaches that can be used to meet these goals; that against opponents who would foil straightforward efforts to win, indirect approaches such as trying to create a fork can produce victories; and that application of more basic processes such as logic and systematic search skills, in the service of the goal hierarchy, can allow the goals to be realized. Together, these types of knowledge seem likely to allow children to play games well, to learn new strategies, and to anticipate which strategies will be useful, even before they try them.

This analysis suggests a broad conclusion: In domains where they possess well-organized understanding, regardless of the type of mechanism that produced that understanding, both older and younger children's learning will generally be effective and efficient. In domains in which they possess poorly organized understanding, regardless of what produced that understanding, both older and younger children's learning will generally be ineffective and inefficient.

Differences between Constraints in Privileged and Nonprivileged Domains

Although substantial similarities unite the operation of constraints in privileged and nonprivileged domains, some differences also are apparent. In this discussion, we focus on two qualities (in addition to the domains in which they operate) that distinguish constraints in privileged and nonprivileged domains.

One difference involves greater variability in constraining knowledge in nonprivileged domains—greater variability across individuals, cultural groups, and historical eras, and greater variability in the amount of constraining knowledge, in the ages when the knowledge is acquired, and in the experiences that lead to its being acquired. In privileged domains, virtually everyone makes the acquisition at about the same age, through similar experience with the world, and to roughly the same (high) degree; in nonprivileged domains, greater variability is present at each step.

With regard to understanding of goal hierarchies, the type of constraining knowledge that seemed critical in the present experiments, the variability can be thought of in terms of two qualities: verdicality and availability. Thinking about these qualities is useful not only for understanding the variability of conceptual understanding but also for thinking about the processes by which understanding is acquired. Veridicality involves the degree to which essential and desirable goals are included in goal sketches, undesirable goals not included, and appropriate distinctions between essential and desirable goals made. Availability involves the range of situations in which the relevant goal sketch is considered and the explicitness with which it can be verbalized. Goal sketches may vary along a continuum, in which lack of any specifically relevant goal sketch is at one extreme and possession of a completely veridical and available sketch at the other. In the present study, the 9-year-olds' goal sketch for tic-tac-toe was toward the veridical and available end of the continuum, since it included both desirable and essential goals and could be explicitly verbalized. The 5-year-olds' goal sketch for addition appeared to be more toward the middle; it included the essential goals, but there was no evidence that it included desirable goals, such as minimizing the amount of counting, and it was not sufficiently available to allow most children to explain their strategy judgments.

Development in many domains can be conceptualized as movement from one end of this continuum to the other. The learning of novices working alone in unfamiliar domains will be minimally constrained. As learners discover strategies or are taught them by more knowledgeable people, they obtain a data base concerning the goals that strategies in the domain must meet. The product of this process is an initial goal sketch. This initial goal sketch can then be used to constrain the generation of new strategies, which, once learned, provide data for inducing other essential or desirable goals of strategies in the domain. For example, in the domain of addition, observing that the sum strategy involves representing each addend and generating a quantitative value for the combined representations may be critical for generating an initial goal sketch that indicates that addition strategies should include such information. The fact that the sum approach is the first approach that children learn and are taught in cultures around the world may be attributable to its illuminating the basic goals of addition so clearly, and thus being especially useful for forming an initial goal sketch for the domain. This initial goal sketch, in turn, may lead children only to try legitimate new strategies and may influence their interpretation and evaluation of more advanced later approaches, such as the min strategy, decomposition, and counting from the first addend.

Thus, as children gain experience in a domain, their goal sketches may generally become increasingly veridical and available, and their problem-solving methods increasingly constrained toward promising possibilities. However, variability in the degree to which individuals encounter the experiences that allow this progression to occur, and variability in the gains individuals derive from a given experience, lead to both the goal sketches themselves and the strategies that they allow to be generated being more variable than the products of constraining mechanisms in evolutionarily privileged domains.

A second difference between constraints on learning in privileged and nonprivileged domains seems to be in the extent of developmental changes in understanding of goal hierarchies. In many privileged domains, such as those involving understanding of gravity, object perception, and face recognition, children's activities meet goals, but the goals are so basic to survival and competent functioning as to be invariant from extremely early in life.[5] In contrast, increases in understanding of goal hierarchies over a period of many years contribute to conceptual development in many nonprivileged domains. In such domains, children often begin by pursuing a limited set of goals and during the course of development generate increasingly complete and veridical knowledge of the goals that useful strategies must meet. This increasing understanding of goals is critical to construction of increasingly effective strategies.

Three types of evidence attest to the critical role of understanding of goals in generating and evaluating new strategies in nonprivileged domains. First, results of Experiment 1 of the present study suggested that children give special status to goal information relevant to arithmetic strategies. The experiment explicitly pitted children's familiarity with procedural components of the novel strategies against their knowledge of the goals that must be met by legitimate strategies in the domain. The illegal strategy, which did not satisfy the addition goal sketch, shared many procedural similarities with the familiar sum strategy. The min strategy, which did satisfy it, shared few procedural similarities with the familiar approach. If information about the goals that a strategy met was not considered substantially more important than information concerning procedural similarity, the illegal strategy should have been judged smarter than the min strategy. Actually, children judged the min strategy to be smarter than the illegal strategy and to be at least as smart as the sum strategy. These judgments indicate that children assigned a special status to whether the novel strategies met the essential goals of addition.

A second line of evidence that attests to the critical role of knowledge of goals for learning in everyday domains comes from recent artificial

[5] This is not to deny that in a number of privileged domains, such as counting and predicability, development occurs for many years. Rather, the claim is that the basic goals served by procedures in these and other privileged domains are understood very early.

intelligence research. In the effort to design programs that learn without many missteps, case-based learning and adaptive planning programs have become increasingly prominent. These types of programs index their experience in a domain according to the hierarchy of subgoals that directed their past actions. When encountering a problem in domains as complex, varied, and "ordinary" as inventing new recipes for Chinese cooking (Hammond, 1990), navigating through the New York City subway (Alterman, 1988), preparing breakfast (Robinson and Kolodner, 1991), and constructing geometry proofs (Anderson, 1993), the programs first check to see whether one of their existing strategies can satisfy all current goals. If situational variation renders the stored strategy partially or completely ineffective, the programs search the current context for alternative actions that together satisfy the same subgoal hierarchy as their existing strategy. Thus, similar to the proposed function of the goal sketch, these programs use the goal hierarchy of the domain to generate new strategies. This allows the programs to adapt to novel situational demands with a minimum of trial and error.

A third line of evidence for the critical role of understanding of goals in generating new strategies comes from studies of analogical reasoning. It has long been known that college students who are unable to identify the goal structure of a problem generally fail to apply previously learned solutions to formally isomorphic problems (e.g., Gick & Holyoak, 1980). More recently, Brown and her colleagues found that in young children's reasoning, understanding of the goals met by a strategy in an analogous situation exceeds even age as a predictor of whether a child will generate an effective problem solving procedure in the new situation. For example, Brown, Kane, and Echols (1986) found that although 5-year-olds generally were more successful than 3-year-olds in transferring previously learned solutions to new, structurally parallel problems, those 3-year-olds who could identify the goal structure of the earlier problems were just as successful as the older children in transferring their knowledge. The developmental difference came in the percentage of children at each age who identified the goal structure.

Together, these three sources of evidence indicate the special importance of understanding of goals in constraining discovery of new strategies. It is critical to recognize, however, that even when such understanding allows accurate evaluation of a strategy, it may not be sufficient to generate that strategy. In the judgment phase of Experiment 2, all 24 children stated that the forking strategy was a smart way to play tic-tac-toe because it created two different ways to win, only one of which could be blocked by one's opponent. It is hard to imagine clearer evidence that they understood the goals that a fork served. Yet, in the strategy-use phase, less than 5 minutes later, 13 of the 24 children failed to use the forking strategy in either full or partial games against the computer. This despite the fact that to set up a fork in the partial games, children had to make only one move—a task that required the same amount of looking ahead as the judgment task on which they had just succeeded. The apparent incongruity between children's understanding and their strategy use underscores our characterization of goal sketches as constraints on strategy discovery rather than as self-contained strategy discovery mechanisms. Knowledge of the goal hierarchy of existing strategies can help focus strategy generation mechanisms on promising possibilities, and can be used to reject illegal strategies that violate essential goals, but they do not guarantee discovery of problem-solving procedures that meet the goals.

The present method of both observing children's strategy use and having them judge the value of strategies that they did or did not use allowed us to draw a number of general conclusions regarding their goal sketches. Collecting additional types of data may allow further, increasingly precise conclusions to be drawn. In cases in which children can make explicit their knowledge about goals, specific questions about the advantages and disadvantages of different strategies may yield such detailed information about particular goal sketches. In cases in which the knowledge of goals is implicit, presenting novel tasks in which meeting both essential and optional goals is difficult may prove revealing of the place of different goals within goal hierarchies. Together with the types of data obtained in the present study, these techniques may allow quite precise assessment of the contents of children's (and adults') goal sketches.

CONCLUSIONS

Children's learning in privileged and nonprivileged domains may be more similar than not. Constraints on learning, rather than being unique to privileged domains, may characterize most learning in entirely mundane domains as well. The key factor determining the degree of constraint appears to be the degree to which understanding of goals and causal relations is veridical and available, rather than whether the domain is evolutionarily important, the age at which learning occurs, or whether understanding is produced by domain-specific or general learning mechanisms. Understanding the key goals that strategies in a domain must meet allows children to accurately evaluate potential strategies even before they use them. This is true even when the strategies being evaluated are more conceptually advanced than the children's own strategies. Such understanding may be what makes it possible for children at times to discover new strategies in nonprivileged as well as privileged domains without engaging in any trial and error.

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